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1. Introduction

In this paper we consider the excitation of surface waves on an unidirectionally conducting screen produced by a phased line source located above the screen and perpendicular to the wire elements. The screen consists of an infinite number of straight, perfectly conducting, parallel wires. The screen is conducting only in the direction of the wire elements. The boundary conditions prescribed on the screen are the following: (1) the electric field component parallel to the wires is zero on both sides of the screen, (2) the component of the electric field normal to the wires is continuous across the screen, and (3) the component of the magnetic field parallel to the wires is continuous across the screen. The source of the electromagnetic field is a phased line source located above the screen and oriented in a direction perpendicular to the wire elements. The phased line source consists of a periodic line current with an electric charge distributed along its length. We determine the complete electromagnetic field exactly and give simple expressions for the scattered far field. We show that surface waves exist and give simple expressions for their amplitudes.

Another principal result is the determination of the magnitude of the complex Poynting vector for the radiated power. We find that the pattern function lies on a cone independent of the presence of the screen and that the cone angle depends only on the phasing of the source. The pattern function at points below the screen is independent of the location of the source above the screen. Also, the pattern function vanishes in the direction of the screen and seems concomittant to the existence of surface waves. Two pattern functions are drawn for typical cases of interest. The power propagated by the surface waves is also determined.

Karp [1] has given a general procedure for solving electromagnetic problems involving screens of the type mentioned above. His method is based on the deduction that the scattered magnetic field components in the direction of the wire elements is zero. He shows that the electromagnetic field can be derived from a single scalar wave function that satisfies a mixed boundary condition on the screen and a jump condition across the screen. This procedure was used by Karal and Karp [2] to study the excitation of surface waves on an unidirectionally conducting screen due to an electric dipole source. The types of surface waves which arise in the present paper were first studied systematically by Rumsey [3], who employed a powerful and general method of his own. Hurd [4] had pointed out similar surface waves on semi-infinite unidirectionally conducting screens, but did not discuss the power flow. Toraldo di Francia [5] was one of the first to work in the general area of unidirectionally conducting screens.

Section 2 contains a mathematical formulation of the problem and presents the exact solution. In Section 3 the scattered far field above and below the screen is found by using a simple procedure employed earlier by the authors. In Section 4 it is shown that surface waves of the usual type exist on the screen. Section 5 contains expressions for the radiated far field above and below the screen. By radiated far field we mean the far field due to the source as well as the far field scattered by the screen. In Section 6 the radiated power is determined, and in

Section 7, the power contained in the surface waves and propagated outward along the screen in the direction of the wire elements is determined. An Appendix, in which the nature of the source is discussed, is also included.

2. Formulation and Solution

Consider a phased line source located above an unidirectionally conducting screen and oriented in a direction perpendicular to the wire elements. The wire elements are taken to be parallel to the y-axis and the screen lies in the x-y plane. The phased line source is oriented in the x-direction along the line $z = z_0$. (See Figure 1.) The source is characterized by a current density

$$(2.1) \qquad \begin{aligned} J_x &= -i\omega\epsilon p e^{i\beta x}\,\delta(y)\,\delta(z-z_0) \\ J_y &= J_z = 0 \end{aligned}$$

and a charge density

$$(2.2) \qquad \rho = -\,i\epsilon p\beta e^{i\beta x}\,\delta(y)\,\delta(z-z_0)$$

Here p is the source strength, β is the phasing constant, ϵ is the permittivity, and ω is the angular frequency. The corresponding electric and magnetic field components (See Appendix) are given by

$$(2.3) \qquad \begin{aligned} E_x^s &= \frac{ip\kappa^2}{4} e^{i\beta x} H_0^{(1)}(\kappa R_-) \\ E_y^s &= -\frac{p\beta}{4} e^{i\beta x} \frac{\partial}{\partial y} H_0^{(1)}(\kappa R_-) \\ E_z^s &= -\frac{p\beta}{4} e^{i\beta x} \frac{\partial}{\partial z} H_0^{(1)}(\kappa R_-) \end{aligned}$$

and

$$H_x^s = 0$$

$$H_y^s = \frac{p\omega\epsilon}{4} e^{i\beta x} \frac{\partial}{\partial z} H_0^{(1)}(\kappa R_-) \tag{2.4}$$

$$E_z^s = - \frac{p\omega\epsilon}{4} e^{i\beta x} \frac{\partial}{\partial y} H_0^{(1)}(\kappa R_-)$$

where

$$\kappa^2 = k^2 - \beta > 0, \tag{2.5}$$

$$R_- = \left[y^2 + (z-z_0)^2\right]^{1/2}. \tag{2.6}$$

Here k is the "free space" propagation constant in the medium. Note that when $\beta = 0$, the charge density is zero and the source becomes a simple line current. The electric and magnetic fields simplify accordingly. We consider the more general case of a variable (or phased) line source since surface waves arise on the screen. When the source is a simple line current element without a charge distribution superimposed along its length, surface waves arise only if the source is skew with respect to the screen. This is the case considered by Seshadri [6].

It has been shown by Karp [1] that problems involving unidirectionally conducting screens may be expressed in terms of a single scalar wave function u. If the total electric and magnetic fields are written as

$$\begin{aligned} \bar{E} &= \bar{e} + \bar{E}^s \\ \bar{H} &= \bar{h} + \bar{H}^s \end{aligned} \tag{2.7}$$

where $\bar{e}$ and $\bar{h}$ are the scattered electric and magnetic fields, and $\bar{E}^s$ and $\bar{H}^s$ are the source (or incident) electric and magnetic fields, then

$$e_x = \frac{1}{i\omega\epsilon} \frac{\partial^2 u}{\partial x \partial y}$$

$$(2.8) \qquad e_y = \frac{1}{i\omega\epsilon} \left(\frac{\partial^2}{\partial y^2} + k^2 \right) u$$

$$e_z = \frac{1}{i\omega\epsilon} \frac{\partial^2 u}{\partial y \partial z}$$

and

$$h_x = \frac{\partial u}{\partial z}$$

$$(2.9) \qquad h_y = 0$$

$$h_z = - \frac{\partial u}{\partial x}$$

where

$$(2.10) \qquad (\nabla^2 + k^2)u = 0.$$

The boundary conditions on the screen are

$$(2.11) \qquad E_y = 0, \qquad z = 0$$

$$(2.12) \qquad [E_x] = 0, \qquad z = 0$$

$$(2.13) \qquad [H_y] = 0, \qquad z = 0$$

where the bracket notation denotes the jump across the screen. In terms of u they become

$$(2.14) \qquad \left(\frac{\partial^2}{\partial y^2} + k^2 \right) u = -i\omega\epsilon\, E_y^s, \qquad z = 0$$

and

$$(2.15) \qquad \frac{\partial^2}{\partial x \partial y} [u] = 0, \qquad z = 0.$$

Substituting into (2.14) from (2.3), we find the particular solution for u is

$$(2.16) \qquad u = \frac{p\beta\omega\epsilon}{2k} \int_{-\infty}^{\infty} \exp\{i\beta x + ik|y-\eta|\} \frac{\partial}{\partial\eta} H_0^{(1)}(\kappa[\eta^2+z_0^2]^{1/2})d\eta,$$

$$z = 0.$$

Rewriting, and integrating by parts, we obtain

$$(2.17) \qquad \begin{aligned} u = {} & \frac{ip\beta\omega\epsilon}{8} \exp\{i\beta x + iky\} \int_{-\infty}^{y} e^{-ik\eta} H_0^{(1)}(\kappa[\eta^2 + z_0^2]^{1/2})\, d\eta \\ & - \frac{ip\beta\omega\epsilon}{8} \exp\{i\beta x - iky\} \int_{y}^{\infty} e^{ik\eta} H_0^{(1)}(\kappa[\eta^2 + z_0^2]^{1/2})d\eta, \end{aligned}$$

$$z = 0.$$

The above result is valid on the screen only.

It is next necessary to construct a wave function valid above and below the screen which reduces to (2.17) on the screen and which possesses the proper behavior at infinity. It may be verified that such a wave function is

$$(2.18) \qquad \begin{aligned} u = {} & \frac{ip\beta\omega\epsilon}{8} \exp\{i\beta x + iky\} \int_{-\infty}^{y} e^{-ik\eta} H_0^{(1)}(\kappa[\eta^2+(z\pm z_0)^2]^{1/2})d\eta \\ & - \frac{ip\beta\omega\epsilon}{8} \exp\{i\beta x - iky\} \int_{y}^{\infty} e^{ik\eta} H_0^{(1)}(\kappa[\eta^2 +(z\pm z_0)^2]^{1/2})d\eta. \end{aligned}$$

The positive sign in the quantity $(z\pm z_0)$ is to be taken above the screen $(z \geq 0)$ and the negative sign is to be taken below $(z \leq 0)$. Note that (2.18) does not possess singularities in the regions of definition. Note also that (2.18) satisfies boundary condition (2.15).

The scattered electric and magnetic fields may now be obtained by substituting (2.18) into (2.8) and (2.9). Since the source electric and magnetic fields are given by (2.3) and (2.4), the total electric and magnetic fields are easily found from (2.7).

3. Scattered Far Field

The scattered far field above and below the screen can be found by using a simple procedure employed by the authors in an earlier paper. (See references given in [2].) The basic argument is as follows. Note that from boundary condition (2.14) and equation (2.3) it follows that the wave function $\left(\frac{\partial^2}{\partial y^2} + k^2\right) u + i\omega\epsilon E_y^s$ is regular for $z \leq 0$ and vanishes on the boundary $z = 0$. The outgoing properties of u and E_y^s at infinity insure that it vanishes identically. For $z \geq 0$ the same argument holds if E_y^s is replaced by its image. Consequently.

$$\left(\frac{\partial^2}{\partial y^2} + k^2\right) u = -i\omega\epsilon \, (E_y^s)_{\text{image}} , \qquad z \geq 0$$

(3.1)

$$\left(\frac{\partial^2}{\partial y^2} + k^2\right) u = -i\omega\epsilon \, E_y^s , \qquad z \leq 0.$$

These results can now be used to find the scattered far field.

For purposes of this problem, we now introduce a cylindrical coordinate system defined by

$$x = x$$

(3.2) $$y = R \cos \Theta$$

$$z = R \sin \Theta$$

These, of course, are not the conventional cylindrical coordinates. In terms of these coordinates the scattered far field due to the source becomes

$$(3.3) \qquad E_y^s \to \frac{p\beta\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \cos\Theta \exp\{i\beta x + i\kappa R - i\kappa z_0 \sin\Theta\}$$

Also

$$(3.4) \qquad (E_y^s)_{\text{image}} \to \frac{p\beta\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \cos\Theta \exp\{i\beta x + i\kappa R + i\kappa z_0 \sin\Theta\}.$$

We next assume the scalar function u has the form

$$(3.5) \qquad u \to m(\Theta)\, e^{i\beta x}\, \frac{e^{i\kappa R}}{\sqrt{R}}$$

where m(Θ) is an unknown function to be determined. If we use the operators

$$\frac{\partial}{\partial x} \to i\beta$$

$$(3.6) \qquad \frac{\partial}{\partial y} \to \cos\Theta \frac{\partial}{\partial R} \to i\kappa \cos\Theta$$

$$\frac{\partial}{\partial z} \to \sin\Theta \frac{\partial}{\partial R} \to i\kappa \sin\Theta$$

and substitute (3.3) through (3.6) into (3.1), we can find the unknown function m(Θ). The expression for u then becomes

$$(3.7) \qquad u \to \frac{\omega\epsilon p\beta\kappa}{4}\, e^{-i5\pi/4}\, \frac{\cos\Theta}{(k-\kappa\cos\Theta)(k+\kappa\cos\Theta)}\sqrt{\frac{2}{\pi\kappa R}} \;\exp\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\Theta\}$$

where the plus sign holds above the screen ($z \geq 0$) and the minus sign below ($z \leq 0$).

The scattered fields are given in terms of u by (2.8) and (2.9). Using the operators given by (3.6), we find

$$e_x \to \frac{p\beta^2\kappa^2}{4} e^{-i3\pi/4} \frac{\cos^2\Theta}{(k-\kappa\cos\Theta)(k+\kappa\cos\Theta)} \sqrt{\frac{2}{\pi\kappa R}} \cdot \exp\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\Theta\}$$

$$e_y \to -\frac{p\beta\kappa}{4} e^{-i3\pi/4} \cos\Theta \sqrt{\frac{2}{\pi\kappa R}} \; e\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\Theta\} \tag{3.8}$$

$$e_z \to \frac{p\beta\kappa^3}{4} e^{-i3\pi/4} \frac{\sin\Theta\cos^2\Theta}{(k-\kappa\cos\Theta)(k+\kappa\cos\Theta)} \sqrt{\frac{2}{\pi\kappa R}} \cdot \exp\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\Theta\}$$

and

$$h_x \to \frac{\omega\epsilon p\beta\kappa^2}{4} e^{-i3\pi/4} \frac{\sin\Theta\cos\Theta}{(k-\kappa\cos\Theta)(k+\kappa\cos\Theta)} \sqrt{\frac{2}{\pi\kappa R}} \cdot \exp\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\Theta\}$$

$$h_y \to 0 \tag{3.9}$$

$$h_z \to -\frac{\omega\epsilon p\beta^2\kappa}{4} e^{-i3\pi/4} \frac{\cos\Theta}{(k-\kappa\cos\Theta)(k+\kappa\cos\Theta)} \sqrt{\frac{2}{\pi\kappa R}} \cdot \exp\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\Theta\}.$$

The same convention regarding signs above and below the screen still holds.

We next determine the cylindrical components of the scattered far field. Note that

$$e_x = e_x$$

$$e_R = e_y \cos\Theta + e_z \sin\Theta \tag{3.10}$$

$$e_\Theta = -e_y \sin\Theta + e_z \cos\Theta$$

and similarly for the magnetic field components. Substituting and combining like terms, we find

$$(3.11)\qquad \begin{aligned} e_R &\to -\frac{p\beta^3\kappa}{4}\, e^{-i3\pi/4} \frac{\cos^2\theta}{(k-\kappa\cos\theta)(k+\kappa\cos\theta)}\sqrt{\frac{2}{\pi\kappa R}} \\ &\quad\cdot\exp\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\theta\} \\ e_\theta &\to \frac{p\beta k^2\kappa}{4}\, e^{-i3\pi/4} \frac{\sin\theta\cos\theta}{(k-\kappa\cos\theta)(k+\kappa\cos\theta)}\sqrt{\frac{2}{\pi\kappa R}} \\ &\quad\cdot\exp\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\theta\} \end{aligned}$$

and

$$(3.12)\qquad \begin{aligned} h_R &\to -\frac{\omega\epsilon p\beta^2\kappa}{4}\, e^{-i3\pi/4} \frac{\sin\theta\cos\theta}{(k-\kappa\cos\theta)(k+\kappa\cos\theta)}\sqrt{\frac{2}{\pi\kappa R}} \\ &\quad\cdot\exp\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\theta\} \\ h_\theta &\to -\frac{\omega\epsilon p\beta^2\kappa}{4}\, e^{-i3\pi/4} \frac{\cos^2\theta}{(k-\kappa\cos\theta)(k+\kappa\cos\theta)}\sqrt{\frac{2}{\pi\kappa R}} \\ &\quad\cdot\exp\{i\beta x + i\kappa R \pm i\kappa z_0 \sin\theta\} \end{aligned}$$

4. Surface Waves

In order to show the existence of surface waves we consider the expression for u given by (2.18) and assume that $|y|$ is large. It is easily shown that

$$(4.1)\qquad u \to \frac{ip\beta\omega\epsilon}{8}\,\operatorname{sgn} y\; e^{i\beta x + ik|y|}\int_{-\infty}^{\infty} e^{-ik\eta}\, H_0^{(1)}\left(\kappa\left[\eta^2+(z\pm z_0)^2\right]^{1/2}\right)d\eta$$

where the upper sign is to be taken above the screen ($z \geq 0$) and the lower sign below screen ($z \leq 0$). The integral appearing in (4.1) is of a standard type (see Noble [7]) and can be evaluated exactly. Hence the expression for u simplifies and may be written as

$$u \rightarrow \frac{p\omega\epsilon}{16} \operatorname{sgn} y \exp\{-\beta|z \pm z_0|\} \exp\{i\beta x + ik|y|\} \tag{4.2}$$

The electric and magnetic fields may be calculated from (2.8) and (2.9). They are

$$\begin{aligned} e_x &\rightarrow \frac{ip\beta k}{16} \operatorname{sgn} y \exp\{-\beta|z \pm z_0|\}\exp\{i\beta x + ik|y|\} \\ e_y &\rightarrow 0 \\ e_x &\rightarrow \mp \frac{p\beta k}{16} \operatorname{sgn} y \exp\{-\beta|z \pm z_0|\}\exp\{i\beta x + ik|y|\} \end{aligned} \tag{4.3}$$

and

$$\begin{aligned} h_x &\rightarrow \mp \frac{p\beta\omega\epsilon}{16} \operatorname{sgn} y \exp\{-\beta|z \pm z_0|\}\exp\{i\beta x + ik|y|\} \\ h_y &\rightarrow 0 \\ h_z &\rightarrow - \frac{ip\beta\omega\epsilon}{16} \operatorname{sgn} y \exp\{-\beta|z \pm z_0|\}\exp\{i\beta x + ik|y|\} \end{aligned} \tag{4.4}$$

The above solutions represent surface waves of the usual type and have been discussed by Rumsey. It is interesting to note, however, that they are different from those obtained by Karal and Karp [2] for an electric dipole source located above a unidirectionally conducting screen. Hence the nature of the excitation is important in determining the type of surface wave launched on the screen.

5. Radiated Far Field

We next determine the radiated far field above and below the screen. In this calculation we take into account the far field due to the source as well as the far field scattered by the screen. The far field due to the phase line source is easily found from (2.3) and (2.4) by using the asymptotic expression for the Hankel function $H_0^{(1)}(\kappa R_-)$, the operators given by (3.6) and the relation $R_- \approx R - z_0 \sin \Theta$. For large R the incident or source field components become

$$E_x^s \to -\frac{p\kappa^2}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \exp\{i\beta x + i\kappa R - i\kappa z_0 \sin\Theta\}$$

$$(5.1) \qquad E_y^s \to \frac{p\beta\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \cos\Theta \exp\{i\beta x + i\kappa R - i\kappa z_0 \sin\Theta\}$$

$$E_z^s \to \frac{p\beta\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \sin\Theta \exp\{i\beta x + i\kappa R - i\kappa z_0 \sin\Theta\}$$

and

$$H_x^s \to 0$$

$$(5.2) \qquad H_y^s \to -\frac{p\omega\epsilon\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \sin\Theta \exp\{i\beta x + i\kappa R - i\kappa z_0 \sin\Theta\}$$

$$H_z^s \to \frac{p\omega\epsilon\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \cos\Theta \exp\{i\beta x + i\kappa R - i\kappa z_0 \sin\Theta\}$$

In terms of the cylindrical components introduced earlier, the incident far field can be written

$$E_x^s \to -\frac{p\kappa^2}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \exp\{i\beta x + i\kappa R - i\kappa z_0 \sin\theta\}$$

$$E_R^s \to \frac{p\beta\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \exp\{i\beta x + i\kappa R - i\kappa z_0 \sin\theta\} \tag{5.3}$$

$$E_\theta^s \to 0$$

and

$$H_x^s \to 0$$

$$H_R^s \to 0 \tag{5.4}$$

$$H_\theta^s \to \frac{p\omega\epsilon\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; ee^{-i3\pi/4} \exp\{i\beta x + i\kappa R - i\kappa z_0 \sin\theta\}$$

The radiated far field is easily found by adding the components of the incident field given above to the components of the scattered field given in Section 3. We find below the screen ($z \leq 0$) that

$$E_x \to -\frac{pk^2\kappa^2}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4}\; \frac{\sin^2\theta}{(k^2-\kappa^2\cos^2\theta)} \exp\{i\beta x+i\kappa R-i\kappa z_0 \sin\theta\}$$

$$E_R \to \frac{pk^2\beta\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4}\; \frac{\sin^2\theta}{(k^2-\kappa^2\cos^2\theta)} \exp\{i\beta x+i\kappa R-i\kappa z_0 \sin\theta\} \tag{5.5}$$

$$E_\theta \to \frac{pk^2\beta\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4}\; \frac{\sin\theta\cos\theta}{(k^2-\kappa^2\cos^2\theta)} \exp\{i\beta x+i\kappa R-i\kappa z_0 \sin\theta\}$$

and

$$H_x \to \frac{\omega\epsilon p\beta\kappa^2}{4} \sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \frac{\sin\theta\cos\theta}{(k^2-\kappa^2\cos^2\theta)} \exp\{i\beta x+i\kappa R-i\kappa z_0 \sin\theta\}$$

$$(5.6) \qquad H_R \to -\frac{\omega\epsilon p\beta^2\kappa}{4} \sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \frac{\sin\theta\cos\theta}{(k^2-\kappa^2\cos^2\theta)} \exp\{i\beta x+i\kappa R-i\kappa z_0 \sin\theta\}$$

$$H_\theta \to \frac{\omega\epsilon p k^2\kappa}{4} \sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4} \frac{\sin^2\theta}{(k^2-\kappa^2\cos^2\theta)} \exp\{i\beta x+i\kappa R-i\kappa z_0 \sin\theta\}$$

Above the screen ($z \geq 0$) we have

$$E_x \to -\frac{p\kappa^2}{4} \sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4}(k^2-\kappa^2\cos^2\theta)^{-1}\Big[k^2\sin^2\theta\cos(\kappa z_0\sin\theta) - i(k^2\sin^2\theta + 2\beta^2\cos^2\theta)\sin(\kappa z_0\sin\theta)\Big]\exp\{i\beta x+i\kappa R\}$$

$$(5.7) \qquad E_R \to \frac{p\beta\kappa}{4} \sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4}(k^2-\kappa^2\cos^2\theta)^{-1}\Big[k^2\sin^2\theta\cos(\kappa z_0\sin\theta) - i(k^2\sin^2\theta + 2\beta^2\cos^2\theta)\sin(\kappa z_0\sin\theta)\Big]\exp\{i\beta x+i\kappa R\}$$

$$E_\theta \to \frac{p\beta k^2\kappa}{4} \sqrt{\frac{2}{\pi\kappa R}}\; e^{-i3\pi/4}(k^2-\kappa^2\cos^2\theta)^{-1}\sin\theta\cos\theta \cdot\Big[\cos(\kappa z_0\sin\theta)+i\sin(\kappa z_0\sin\theta)\Big]\exp\{i\beta x+i\kappa R\}$$

and

$$H_x \to \frac{\omega\epsilon p\beta\kappa^2}{4}\sqrt{\frac{2}{\pi\kappa R}}\, e^{-i3\pi/4}\,(k^2-\kappa^2\cos^2\Theta)^{-1}\sin\Theta\cos\Theta$$

$$\cdot\left[\cos(\kappa z_0\sin\Theta)+i\sin(\kappa z_0\sin\Theta)\right]\exp\{i\beta x+i\kappa R\}$$

(5.8)
$$H_R \to -\frac{\omega\epsilon p\beta^2\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\, e^{-i3\pi/4}\,(k^2-\kappa^2\cos^2\Theta)^{-1}\sin\Theta\cos\Theta$$

$$\cdot\left[\cos(\kappa z_0\sin\Theta)+i\sin(\kappa z_0\sin\Theta)\right]\exp\{i\beta x+i\kappa R\}$$

$$H_\Theta \to \frac{\omega\epsilon p\kappa}{4}\sqrt{\frac{2}{\pi\kappa R}}\, e^{-i3\pi/4}\,(k^2-\kappa^2\cos^2\Theta)^{-1}\left[k^2\sin^2\Theta\cos(\kappa z_0\sin\Theta)\right.$$

$$\left.-\,i(k^2\sin^2\Theta+2\beta^2\cos^2\Theta)\sin(\kappa z_0\sin\Theta)\right]\exp\{i\beta x+i\kappa R\}$$

Note that the components of the far field vanish when $\sin\Theta = 0$. This is the direction in which the surface wave exists and takes over the propagation of power. A similar situation occurs for the far field amplitude along an impedance structure, as has been pointed out by the authors [8]. This fact is helpful for treating problems with terminations [9].

6. Radiated Power

The complex Poynting vector is given by

(6.1)
$$\bar{S} = \frac{1}{2}\,\mathrm{Re}\left[\bar{E}\times\bar{H}^*\right]$$

where the asterisk denotes the complex conjugate of the magnetic field. The cylindrical components of $\bar{S}$ are easily found by using the fields given in Section 5. We find that

(6.2)

$$S_x = \frac{\omega \epsilon p^2 \beta \kappa}{16\pi R} \frac{\left[k^2 \sin^2\theta + 4\beta^2 \cos^2\theta \sin^2(\kappa z_0 \sin\theta)\right]}{(k^2 - \kappa^2 \cos^2\theta)}, \quad z \geq 0$$

$$S_x = \frac{\omega \epsilon p^2 k^2 \beta \kappa}{16\pi R} \frac{\sin^2\theta}{(k^2 - \kappa^2 \cos^2\theta)}, \quad z \leq 0$$

(6.3)

$$S_R = \frac{\omega \epsilon p^2 \kappa^2}{16\pi R} \frac{\left[k^2 \sin^2\theta + 4\beta^2 \cos^2\theta \sin^2(\kappa z_0 \sin\theta)\right]}{(k^2 - \kappa^2 \cos^2\theta)}, \quad z \geq 0$$

$$S_R = \frac{\omega \epsilon p^2 k^2 \kappa^2}{16\pi R} \frac{\sin^2\theta}{(k^2 - \kappa^2 \cos^2\theta)}, \quad z \leq 0$$

and

(6.4) $S_\theta = 0$

The magnitude of power flow $S = \sqrt{S_x^2 + S_R^2}$ is given by

(6.5) $$S = \frac{\omega \epsilon p^2 k^2}{16\pi R} \sqrt{1 - \left(\frac{\beta}{k}\right)^2} \, F(\theta)$$

where $F(\theta)$ is the pattern function except for the factor $\sqrt{1 - \left(\frac{\beta}{k}\right)^2}$.

Below the screen

(6.6) $$F(\theta) = \frac{1}{1 + \left(\frac{\beta}{k}\right)^2 \cot^2\theta}, \quad z \leq 0$$

and above the screen

(6.7) $$F(\theta) = \frac{1}{1 + \left(\frac{\beta}{k}\right)^2 \cot^2\theta} \left[1 + 4\left(\frac{\beta}{k}\right)^2 \cot^2\theta \sin^2\left(kz_0 \sqrt{1 - \left(\frac{\beta}{k}\right)^2} \sin\theta\right)\right],$$

$$z \geq 0.$$

Note that the pattern function is symmetrical when the source lies on the screen ($z_0 = 0$). Note also that the pattern function is zero on the screen regardless of the location of the source. When this occurs, however, the surface wave exists and takes over the propagation of power. Also

$$(6.8) \qquad \frac{S_R}{S_x} = \frac{\kappa}{\beta} = \sqrt{\left(\frac{k}{\beta}\right)^2 - 1} = \tan\psi .$$

Here ψ is the angle between S_R and S_x and gives the direction of power flow. Hence, in general, the pattern function lies on a cone while for the special case $\beta = 0$ we have the usual case of a pattern function on a plane. The cone of power is the same as in the case of a phased line source without a screen (See Felsen and Marcuvitz [10].) despite the more complicated form of the functions S_x and S_R above and below the screen. In Figure 2 we plot the pattern function $F(\Theta)$ for three different values of $\frac{\beta}{k}$ when the source lies on the screen. The value $\frac{\beta}{k} = 1$ is a limiting case. Another limiting case is the value $\frac{\beta}{k} = 0$ and this yields a circle of unit radius. Note that $F(\Theta)$ should be multiplied by the factor $\sqrt{1 - \left(\frac{\beta}{k}\right)^2}$ if relative magnitudes are desired. In Figure 3 we plot the pattern function $F(\Theta)$ for three different values of kz_0 when the value of $\frac{\beta}{k} = \frac{1}{2}$. Note that the pattern function beneath the screen is unaffected by the location of the phased line source.

The power radiated in the radial direction per unit length (along the x-axis) is given by

(6.9)
$$P_R = \int_0^{\pi} (S_R)_{z \geq 0} \, R d\theta + \int_{\pi}^{2\pi} (S_R)_{z \leq 0} \, R d\theta .$$

Substituting from (7.3) the above becomes

(6.10)
$$P_R = \frac{\omega \epsilon p^2 \kappa^2 k^2}{8\pi} \int_0^{\pi} \frac{\sin^2\theta \, d\theta}{(k^2 - \kappa^2 \cos^2\theta)} + \frac{\omega \epsilon p^2 \kappa^2 \beta^2}{4\pi} \int_0^{\pi} \frac{\cos^2\theta \sin^2(\kappa z_0 \sin\theta) d\theta}{(k^2 - \kappa^2 \cos^2\theta)} .$$

Similarly, the power radiated in the positive x-direction through a radial band of unit thickness is given by

(6.11)
$$P_x = \int_0^{\pi} (S_x)_{z \geq 0} \, R d\theta + \int_{\pi}^{2\pi} (S_x)_{z \leq 0} \, R d\theta .$$

Substituting from (7.2), the above becomes

(6.12)
$$P_x = \frac{\omega \epsilon p^2 \beta \kappa k^2}{8\pi} \int_0^{\pi} \frac{\sin^2 \theta \, d\theta}{(k^2 - \kappa^2 \cos^2\theta)} + \frac{\omega \epsilon p^2 \beta^3 \kappa}{4\pi} \int_0^{\pi} \frac{\cos^2\theta \sin^2(\kappa z_0 \sin\theta) d\theta}{(k^2 - \kappa^2 \cos^2\theta)} .$$

The power radiated in the negative x-direction is the same except for a difference in sign. The results for P_R and P_x can be simplified somewhat by using elementary trigonometrical formulas. It is easily shown that

(6.13)
$$P_R = \frac{\omega \epsilon p^2 \kappa^2}{8} - \frac{\omega \epsilon p^2 \kappa^2 \beta^2}{8\pi} \int_0^{\pi} \frac{\cos^2\theta \cos(2\kappa z_0 \sin\theta) d\theta}{(k^2 - \kappa^2 \cos^2\theta)} .$$

and

(6.14)
$$P_x = \frac{\omega \epsilon p^2 \beta \kappa}{8} - \frac{\omega \epsilon p^2 \beta^3 \kappa}{8\pi} \int_0^{\pi} \frac{\cos^2\theta \cos(2\kappa z_0 \sin\theta) d\theta}{(k^2 - \kappa^2 \cos^2\theta)} .$$

In these results the first term is the power due to the source alone. Simplifying further we have

$$(6.15)\qquad P_R = \frac{\omega\epsilon p^2\kappa^2}{8}\left[1 + \frac{\beta^2}{\kappa^2}J_0(2\kappa z_0) - \frac{\beta^2 k^2}{\pi\kappa^2}\int_0^\pi \frac{\cos(2\kappa z_0 \sin\theta)d\theta}{(k^2-\kappa^2\cos^2\theta)}\right]$$

and

$$(6.16)\qquad P_x = \frac{\omega\epsilon p^2\beta\kappa}{8}\left[1 + \frac{\beta^2}{\kappa^2}J_0(2\kappa z_0) - \frac{\beta^2 k^2}{\pi\kappa^2}\int_0^\pi \frac{\cos(2\kappa z_0 \sin\theta)d\theta}{(k^2-\kappa^2\cos^2\theta)}\right].$$

Note that P_R/P_x is a simple constant given by κ/β just as if the screen were absent. Here $J_0(2\kappa z_0)$ is the usual Bessel function of the first kind. When the line source lies on the screen, the above results simplify. When $z_0 = 0$ we have

$$(6.17)\qquad P_R = \frac{\omega\epsilon p^2\kappa^2}{8}\left[1 - \frac{\beta}{k+\beta}\right]$$

and

$$(6.18)\qquad P_x = \frac{\omega\epsilon p^2\beta\kappa}{8}\left[1 - \frac{\beta}{k+\beta}\right].$$

The first terms in the brackets of the above expressions represent the power due to the source alone. The second terms represent the effect of the unidirectionally conducting screen. Note that when there is no phasing along the line source (β=0) the power expressions reduce to those for a line source in free space. Hence the presence of the screen does not interact with the applied field when the source lies on the screen and perpendicular to the wire elements.

7. Surface Wave Power

The power contained in the surface waves and propagated outward along the screen in the direction of the wire elements is easily determined. We first calculate the rectangular components of the complex Poynting vector. Making use of (6.1), (4.3) and (4.4), we find

$$S_x = 0$$

$$(7.1) \qquad S_y = \frac{\omega \epsilon p^2 \beta^2 k}{(16)^2} \; e^{-2\beta |z \pm z_0|}$$

$$S_z = 0$$

where the plus sign is to be used above the screen ($z \geq 0$) and the minus sign below ($z \leq 0$).

The power propagated in the positive y-direction per unit length (along the x-axis) is

$$(7.2) \qquad P_y = \int_0^{\infty} (S_y)_{z \geq 0} \, dz + \int_{-\infty}^{0} (S_y)_{z \leq 0} \, dz.$$

Substituting from (8.1) and simplifying, we have

$$(7.3) \qquad P_y = \frac{\omega \epsilon p^2 \beta k}{(16)^2} \; e^{-2\beta z_0}$$

The total surface wave power is the sum of the power propagated outward in the positive y-direction plus the power propagated in the negative y-direction. By symmetry, these are the same and hence the total surface power is twice the result given by (7.3). Thus

$$P_s = \frac{2\omega\epsilon p^2\beta k}{(16)^2} e^{-2\beta z_0}. \tag{7.4}$$

The total surface wave power is greatest when the source lies on the screen ($z_0 = 0$). Also note that when the source is unphased ($\beta = 0$), the surface wave power is zero. Hence phasing is necessary in order to excite surface waves.

Appendix

The electromagnetic field can be derived from a single source function by means of the Hertz vector $\bar{\Pi}$. Let the current and charge densities be expressed in terms of the vector $\bar{P}$ defined by

$$\bar{J} = -i\omega\bar{P} \tag{A.1}$$

$$\rho = -\nabla\cdot\bar{P} \tag{A.2}$$

Then the equation of continuity is satisfied identically and the field equations are

$$\nabla \times \bar{E} - i\omega\mu\bar{H} = 0 \tag{A.3}$$

$$\nabla \times \bar{H} + i\omega\epsilon\bar{E} = -i\omega\bar{P} \tag{A.4}$$

$$\nabla \cdot \bar{H} = 0 \tag{A.5}$$

$$\nabla \cdot \bar{E} = -\frac{1}{\epsilon}\nabla \cdot \bar{P}. \tag{A.6}$$

The above equations are satisfied by

$$\bar{E} = \nabla\nabla \cdot \bar{\Pi} + \omega^2\mu\epsilon\bar{\Pi} \tag{A.7}$$

$$\bar{H} = -i\omega\epsilon\,\nabla \times \bar{\Pi} \tag{A.8}$$

where $\bar{\Pi}$ is any solution of the equation

$$\nabla \times \nabla \times \bar{\Pi} - \nabla\nabla \cdot \bar{\Pi} + \omega^2\mu\epsilon\,\bar{\Pi} = \frac{1}{\epsilon}\bar{P}. \tag{A.9}$$

The rectangular components of $\bar{\Pi}$ satisfy the inhomogeneous wave equation

$$\nabla^2\bar{\Pi} + \omega^2\mu\epsilon\,\bar{\Pi} = -\frac{1}{\epsilon}\bar{P}. \tag{A.10}$$

Suppose

$$\bar{P} = \epsilon p e^{i\beta x}\,\delta(y-y_0)\delta(z-z_0)\mathbf{t}_x. \tag{A.11}$$

Then

$$\Pi_x = \frac{ip}{4} e^{i\beta x} H_0^{(1)} (\kappa R_-) \tag{A.12}$$

$$\Pi_y = \Pi_z = 0 \tag{A.13}$$

where $\kappa^2 = k^2 - \beta^2$ and $R_- = \left[(y-y_0)^2 + (z-z_0)^2\right]^{1/2}$.

From (A.7), (A.8), (A.12) and (A.13) the electromagnetic field is easily determined. We have

$$\begin{aligned} E_x &= \frac{ip\kappa^2}{4} e^{i\beta x} H_0^{(1)} (\kappa R_-) \\ E_y &= -\frac{p\beta}{4} e^{i\beta x} \frac{\partial}{\partial y} H_0^{(1)} (\kappa R_-) \\ E_z &= -\frac{p\beta}{4} e^{i\beta x} \frac{\partial}{\partial z} H_0^{(1)} (\kappa R_-) \end{aligned} \tag{A.14}$$

and

$$\begin{aligned} H_x &= 0 \\ H_y &= \frac{p\omega\epsilon}{4} e^{i\beta x} \frac{\partial}{\partial z} H_0^{(1)} (\kappa R_-) \\ H_z &= -\frac{p\omega\epsilon}{4} e^{i\beta x} \frac{\partial}{\partial y} H_0^{(1)} (\kappa R_-) \end{aligned} \tag{A.15}$$

We now determine the nature of the source by using (A.1), (A.2) and (A.11). The current density is given by

$$\begin{aligned} J_x &= -i\omega\epsilon p e^{i\beta x} \delta(y-y_0)\delta(z-z_0) \\ J_y &= 0 \\ J_z &= 0 \end{aligned} \tag{A.16}$$

The charge density is given by

$$(A.17) \qquad \rho = -ip\beta\epsilon e^{i\beta x}\,\delta(y-y_0)\delta(z-z_0)$$

Hence the source is due to a variable (or phased) line current with a concomittant variable (or phased) electric charge superimposed along its length.

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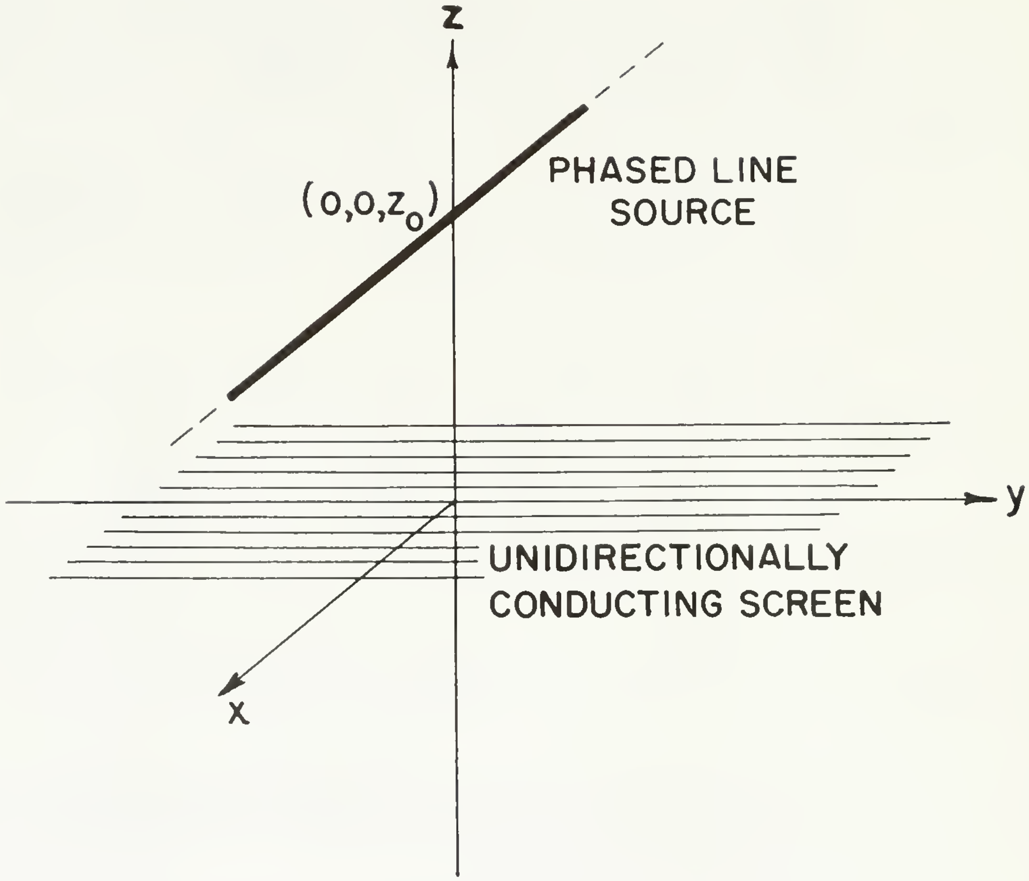

Figure 1. Unidirectionally Conducting Screen Excited by a Phased Line Source.

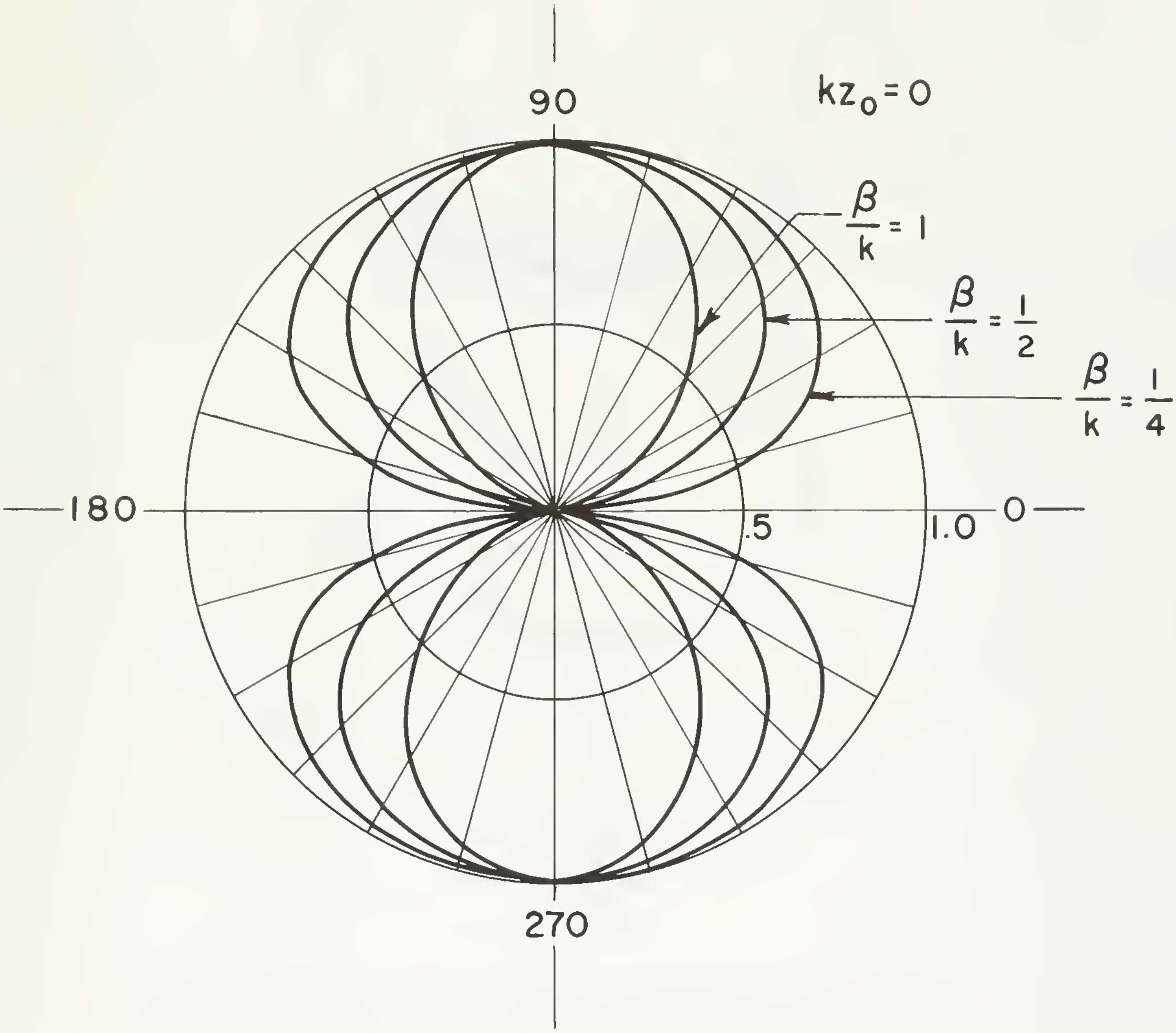

Figure 2. Magnitude of the complex Poynting vector when the source lies on the screen for three different values of the phasing ratio $\frac{\beta}{k}$. When $\frac{\beta}{k} = 0$, the pattern function is a circle of unit radius. Note: the pattern function should be multiplied by the factor $\left[1 - \left(\frac{\beta}{k}\right)^2\right]^{1/2}$ if relative magnitudes are desired.

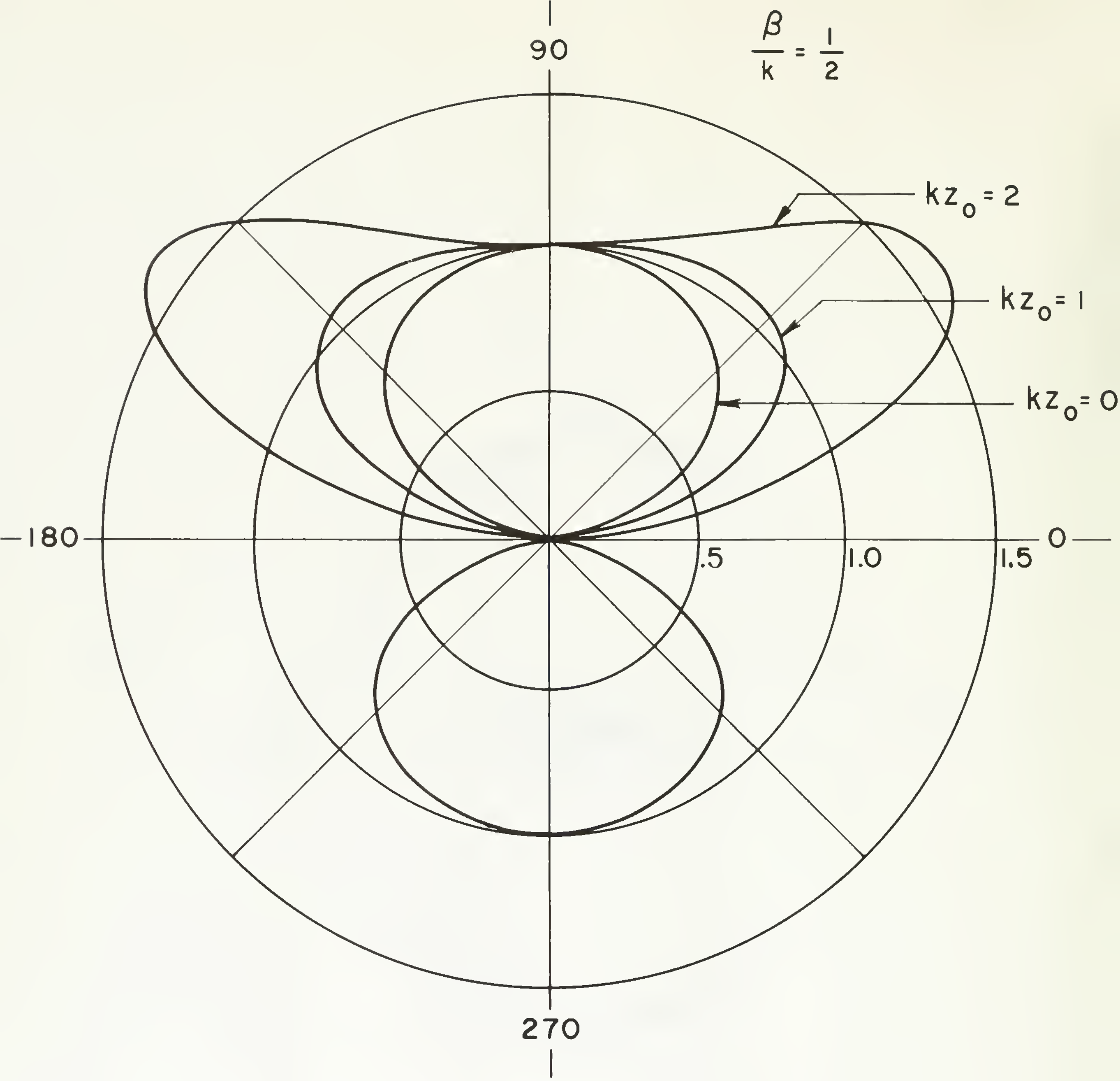

Figure 3. Magnitude of the complex Poynting vector when the phasing ratio is constant $\left(\frac{\beta}{k} = \frac{1}{2}\right)$ and the position of the source is changed. The pattern function below the screen is unaffected by the location of the source. Note: the pattern function should be multiplied by the factor $\left[1 - \left(\frac{\beta}{k}\right)^2\right]^{1/2}$ if relative magnitudes are desired.

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